**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

* Answer: - Z-Scores = (50-45)/8

= 0.625

probability P(X>50) = 0.26598552904870054

probability P(X<=50) = 0.7340144709512995

P(X>50) = 1 - P(X<=50) = 1 – 0.734

= 0.266

So, B is the right Answer

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

* Answer: -

P (X>44) = 0.15865525393145707

P(Between 38 and 44) = 0.3413447460685429

The Statement is False because as values calculated shows that employees between 38 and 44 years of age is more.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

* Answer: -

The above statement is true as per the calculation

Please refer .ipynb file for calculation

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

* Answer: -

According to the Central Limit Theorem, any large sum of independent, identically distributed random variables is Normal.

So, given X1~ N (μ, σ2) and *X*2 ~ N (μ, σ2) are two independent identically distributed random variables.

The sum of normal random variables is given by = X + Y ~ N (μ1 + μ2, σ21 + σ22)

The difference of normal random variables is given by = Z ~ N (μ1 - μ2, σ21 + σ22)

When Z = aX the product of X is given by = Z ~ N (aμ1 + bμ2, a2σ21 +b2σ22)

Given to find 2X1

Thus, following the property of multiplication, we get

2X1 ~ N (2μ, 22 σ2) => 2X1 ~ N (2μ, 4σ2)

and the property of addition = 2X1 – (X1 + X2) ~ N (2μ - 2μ, 2σ21 + 4σ21) ~ **N (0, 6σ2)**

The mean of 2X1 and X1 + X2 is same but the var(σ2) of 2X1 is 2 times more than the variance of X1 + X2

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

* Answer: -

The probability of getting value between a and b is 0.99

So, the probability of getting value outside a and b is 1 – 0.99 = 0.01

The probability towards left of a = 0.01/2 = 0.05

The probability towards right of b = 0.01/2 = 0.05

Since we have probability of a and b, we need calculate between the probability of X – the random variable at a and b which has these probabilities

By finding Standard Normal Variable (z), need to be calculated X: - Z = (X – Mue) / Sigma

For a probability of 0.005, z values is -2.57

Z \* σ + μ = x

-(-2.57) \* 20 \* 100 = **151.4**

-(-2.57) \* 20 + 100 = **48.6**

**Option D is Correct**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

* Answer: - Range is Rs (99.00810347848784, 980.9918965215122)

1. Specify the 5th percentile of profit (in Rupees) for the company

* Answer: - 5th percentile of profit (in Million Rupees) is 170.0

1. Which of the two divisions has a larger probability of making a loss in a given year?

* Answer: - 0.0477903522728147

Refer .ipynb file for answer